

Question 4

Evaluate the integral

$$\int x \sqrt{x^2 + 4} \ln(x^2 + 4) dx.$$

Question 4 - Solution

Use substitution. Let

$$u = x^2 + 4, \quad du = 2x \, dx.$$

Then

$$\int x\sqrt{x^2 + 4} \ln(x^2 + 4) \, dx = \frac{1}{2} \int u^{1/2} \ln u \, du.$$

Apply integration by parts with

$$w = \ln u, \quad dv = u^{1/2} \, du.$$

Then

$$dw = \frac{1}{u} \, du, \quad v = \frac{2}{3} u^{3/2}.$$

Thus,

$$\int u^{1/2} \ln u \, du = \frac{2}{3} u^{3/2} \ln u - \frac{2}{3} \int u^{1/2} \, du = \frac{2}{3} u^{3/2} \ln u - \frac{4}{9} u^{3/2}.$$

Multiply by $\frac{1}{2}$ and substitute back:

$$\frac{1}{3} (x^2 + 4)^{3/2} \ln(x^2 + 4) - \frac{2}{9} (x^2 + 4)^{3/2} + C$$