

Question 3

Evaluate the integral

$$\int \frac{x^3}{(x^2 + 1)^2} dx.$$

Question 3 - Solution

Rewrite the integrand by separating powers of x :

$$\frac{x^3}{(x^2 + 1)^2} = \frac{x(x^2 + 1 - 1)}{(x^2 + 1)^2} = \frac{x}{x^2 + 1} - \frac{x}{(x^2 + 1)^2}.$$

Split the integral:

$$\int \frac{x}{x^2 + 1} dx - \int \frac{x}{(x^2 + 1)^2} dx.$$

For the first integral, let $u = x^2 + 1$, $du = 2x dx$:

$$\int \frac{x}{x^2 + 1} dx = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln(x^2 + 1).$$

For the second integral, use the same substitution:

$$\int \frac{x}{(x^2 + 1)^2} dx = \frac{1}{2} \int u^{-2} du = -\frac{1}{2u} = -\frac{1}{2(x^2 + 1)}.$$

Combine results:

$$\boxed{\frac{1}{2} \ln(x^2 + 1) + \frac{1}{2(x^2 + 1)} + C}$$