

Question 2

Evaluate the integral

$$\int x^2 \ln(x^2 + 1) dx.$$

Question 2 - Solution

Use integration by parts. Let

$$u = \ln(x^2 + 1), \quad dv = x^2 dx.$$

Then

$$du = \frac{2x}{x^2 + 1} dx, \quad v = \frac{x^3}{3}.$$

Apply integration by parts:

$$\int x^2 \ln(x^2 + 1) dx = \frac{x^3}{3} \ln(x^2 + 1) - \frac{2}{3} \int \frac{x^4}{x^2 + 1} dx.$$

Simplify the integrand by division:

$$\frac{x^4}{x^2 + 1} = x^2 - 1 + \frac{1}{x^2 + 1}.$$

Thus,

$$\int \frac{x^4}{x^2 + 1} dx = \int (x^2 - 1) dx + \int \frac{1}{x^2 + 1} dx = \frac{x^3}{3} - x + \arctan x.$$

Substitute back:

$$\int x^2 \ln(x^2 + 1) dx = \frac{x^3}{3} \ln(x^2 + 1) - \frac{2}{3} \left(\frac{x^3}{3} - x + \arctan x \right).$$

Final Answer:

$$\frac{x^3}{3} \ln(x^2 + 1) - \frac{2x^3}{9} + \frac{2x}{3} - \frac{2}{3} \arctan x + C$$