

Question 9

Find the general antiderivative of

$$\int \frac{\arctan x}{x^2} dx.$$

Question 9 - Solution

We use integration by parts.

Let

$$u = \arctan x, \quad dv = \frac{1}{x^2} dx.$$

Then

$$du = \frac{1}{1+x^2} dx, \quad v = -\frac{1}{x}.$$

Applying integration by parts,

$$\int \frac{\arctan x}{x^2} dx = -\frac{\arctan x}{x} + \int \frac{1}{x(1+x^2)} dx.$$

Now decompose the integrand:

$$\frac{1}{x(1+x^2)} = \frac{1}{x} - \frac{x}{1+x^2}.$$

Thus,

$$\int \frac{1}{x(1+x^2)} dx = \ln|x| - \frac{1}{2} \ln(1+x^2).$$

Final Answer:

$$-\frac{\arctan x}{x} + \ln|x| - \frac{1}{2} \ln(1+x^2) + C$$