

Question 7

Find an antiderivative of

$$\int \left(\frac{\sqrt{1+2x}}{x} - \frac{1}{x\sqrt{1+2x}} \right) dx.$$

Question 7 - Solution

Combine the terms into a single fraction:

$$\frac{\sqrt{1+2x}}{x} - \frac{1}{x\sqrt{1+2x}} = \frac{(1+2x) - 1}{x\sqrt{1+2x}} = \frac{2x}{x\sqrt{1+2x}} = \frac{2}{\sqrt{1+2x}}.$$

Thus the integral simplifies to

$$\int \frac{2}{\sqrt{1+2x}} dx.$$

Let

$$u = 1 + 2x, \quad du = 2 dx.$$

Then

$$\int \frac{2}{\sqrt{1+2x}} dx = \int u^{-1/2} du = 2u^{1/2}.$$

Substitute back:

$$\boxed{2\sqrt{1+2x} + C}$$