

Question 6

Find the general antiderivative of

$$\int \left(x e^{-x^2} + \ln(1+x) \right) dx.$$

Question 6 - Solution

We evaluate each term separately.

First term:

$$\int x e^{-x^2} dx.$$

Let $u = -x^2$, so $du = -2x dx$:

$$\int x e^{-x^2} dx = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^{-x^2}.$$

Second term:

$$\int \ln(1+x) dx.$$

Use integration by parts with

$$u = \ln(1+x), \quad dv = dx.$$

Then

$$du = \frac{1}{1+x} dx, \quad v = x.$$

Thus,

$$\int \ln(1+x) dx = x \ln(1+x) - \int \frac{x}{1+x} dx.$$

Rewrite the integrand:

$$\frac{x}{1+x} = 1 - \frac{1}{1+x}.$$

Hence,

$$\int \frac{x}{1+x} dx = x - \ln(1+x).$$

Combining results:

$$-\frac{1}{2} e^{-x^2} + x \ln(1+x) - x + \ln(1+x) + C$$