

Question 4

Evaluate the integral

$$\int \left(\sqrt{x} \ln x + \frac{1}{\sqrt{x}} \right) dx.$$

Question 4 - Solution

We first rewrite the integrand using exponents:

$$\sqrt{x} \ln x + \frac{1}{\sqrt{x}} = x^{1/2} \ln x + x^{-1/2}.$$

For the first term, apply integration by parts with

$$u = \ln x, \quad dv = x^{1/2} dx.$$

Then

$$du = \frac{1}{x} dx, \quad v = \frac{2}{3} x^{3/2}.$$

Thus,

$$\int x^{1/2} \ln x dx = \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int x^{1/2} dx = \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2}.$$

For the second term,

$$\int x^{-1/2} dx = 2x^{1/2}.$$

Combining results:

$$\boxed{\frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + 2x^{1/2} + C}$$