

**Question 3**

Determine an antiderivative of

$$\int \frac{x^3}{\sqrt{1-x^2}} dx.$$

**Question 3 - Solution**

For  $|x| < 1$ , set  $u = 1 - x^2$ , so  $du = -2x \, dx$  and  $x^2 = 1 - u$ .

$$\int \frac{x^3}{\sqrt{1-x^2}} \, dx = -\frac{1}{2} \int (1-u)u^{-1/2} \, du = -\sqrt{u} + \frac{1}{3}u^{3/2} + C.$$

$$-\sqrt{1-x^2} + \frac{1}{3}(1-x^2)^{3/2} + C.$$

Differentiation gives  $x/\sqrt{1-x^2} - x\sqrt{1-x^2} = x^3/\sqrt{1-x^2}$ .