

Question 1

Evaluate the indefinite integral

$$\int \left(x^2 e^{x^3} + \frac{1}{x} \sqrt{1 + \ln x} - \frac{3x^2}{1 + x^3} \right) dx.$$

Question 1 - Solution

We integrate term by term.

First term:

$$\int x^2 e^{x^3} dx.$$

Let $u = x^3$, so $du = 3x^2 dx$:

$$\int x^2 e^{x^3} dx = \frac{1}{3} \int e^u du = \frac{1}{3} e^{x^3}.$$

Second term:

$$\int \frac{1}{x} \sqrt{1 + \ln x} dx.$$

Let $u = 1 + \ln x$, so $du = \frac{1}{x} dx$:

$$\int \sqrt{u} du = \frac{2}{3} u^{3/2} = \frac{2}{3} (1 + \ln x)^{3/2}.$$

Third term:

$$\int \frac{3x^2}{1 + x^3} dx.$$

Let $u = 1 + x^3$, so $du = 3x^2 dx$:

$$\int \frac{3x^2}{1 + x^3} dx = \int \frac{du}{u} = \ln(1 + x^3).$$

Final Answer:

$$\frac{1}{3} e^{x^3} + \frac{2}{3} (1 + \ln x)^{3/2} - \ln(1 + x^3) + C$$