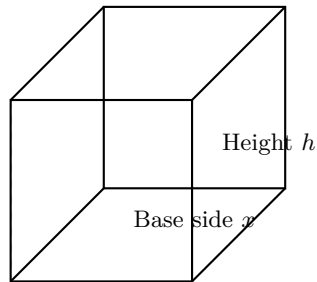


Question 10

A box with a square base and an open top must have a volume of V cubic units. Find the dimensions of the box that minimize its surface area.



Question 10 - Solution

Let: - x = length of the base side (square base) - h = height of the box - Volume: $V = x^2h \Rightarrow h = \frac{V}{x^2}$

Surface Area (SA): The box has no top, so:

$$A(x) = \text{Base} + 4 \text{ sides} = x^2 + 4xh$$

$$A(x) = x^2 + 4x \left(\frac{V}{x^2} \right) = x^2 + \frac{4V}{x}$$

Minimize $A(x)$:

Differentiate:

$$A'(x) = 2x - \frac{4V}{x^2}$$

Set $A'(x) = 0$:

$$2x - \frac{4V}{x^2} = 0 \Rightarrow 2x^3 = 4V \Rightarrow x^3 = 2V \Rightarrow x = \sqrt[3]{2V}$$

Then:

$$h = \frac{V}{x^2} = \frac{V}{\left(\sqrt[3]{2V}\right)^2} = \frac{V}{\sqrt[3]{4V^2}} = \sqrt[3]{\frac{V}{4}}$$

Answer:

$x = \sqrt[3]{2V}, \quad h = \sqrt[3]{\frac{V}{4}}$

These dimensions give the minimum surface area for the box.