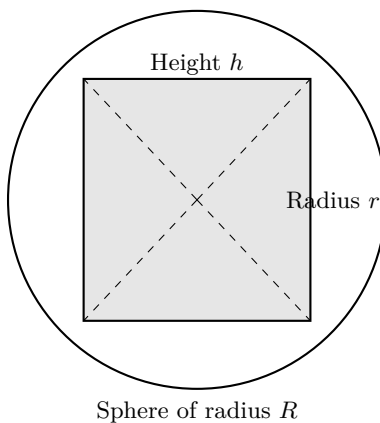


Question 9

A cylinder is inscribed in a sphere of radius R . Find the dimensions (radius and height) of the cylinder that has the maximum volume.



Question 9 - Solution

Let the cylinder have: - Radius: r - Height: h

It is inscribed in a sphere of radius R , so by the Pythagorean Theorem applied to the right triangle formed from the center of the sphere to the top edge of the cylinder:

$$r^2 + \left(\frac{h}{2}\right)^2 = R^2$$

Volume of the cylinder:

$$V = \pi r^2 h$$

From the constraint:

$$r^2 = R^2 - \left(\frac{h}{2}\right)^2 = R^2 - \frac{h^2}{4}$$

Substitute into the volume:

$$V(h) = \pi \left(R^2 - \frac{h^2}{4}\right) h = \pi \left(R^2 h - \frac{h^3}{4}\right)$$

Differentiate and maximize:

$$V'(h) = \pi \left(R^2 - \frac{3h^2}{4}\right)$$

Set $V'(h) = 0$:

$$R^2 - \frac{3h^2}{4} = 0 \Rightarrow h^2 = \frac{4R^2}{3} \Rightarrow h = \frac{2R}{\sqrt{3}}$$

Then:

$$r^2 = R^2 - \left(\frac{h}{2}\right)^2 = R^2 - \frac{1}{4} \cdot \frac{4R^2}{3} = R^2 - \frac{R^2}{3} = \frac{2R^2}{3} \Rightarrow r = \sqrt{\frac{2}{3}}R$$

Answer:

$$\boxed{h = \frac{2R}{\sqrt{3}}, \quad r = \sqrt{\frac{2}{3}}R}$$

These are the dimensions of the inscribed cylinder with maximum volume.