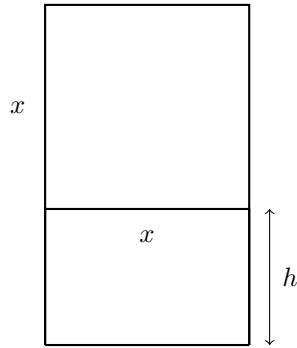


Question 7

A rectangular box with a square base and no top is to have a volume of 500 cm^3 . What dimensions will minimize the surface area?



Open-top square box

Question 7 - Solution

Let: - x = side length of the square base (in cm) - h = height of the box (in cm)

Volume constraint:

$$V = x^2h = 500 \Rightarrow h = \frac{500}{x^2}$$

Surface area to minimize: - One bottom square: x^2 - Four sides: $4(xh)$

$$A(x) = x^2 + 4xh = x^2 + 4x \cdot \frac{500}{x^2} = x^2 + \frac{2000}{x}$$

Take derivative:

$$A'(x) = 2x - \frac{2000}{x^2}$$

Set derivative to zero:

$$2x = \frac{2000}{x^2} \Rightarrow 2x^3 = 2000 \Rightarrow x^3 = 1000 \Rightarrow x = 10$$

Find h :

$$h = \frac{500}{x^2} = \frac{500}{100} = 5$$

Answer:

$x = 10 \text{ cm}, \quad h = 5 \text{ cm}$

These dimensions minimize the surface area.