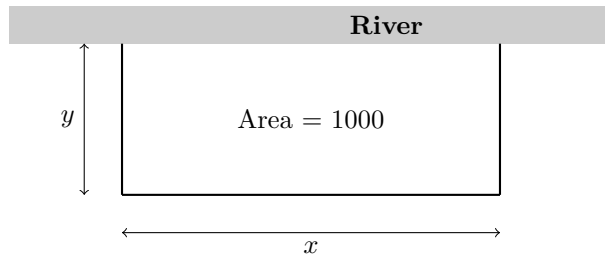


Question 6

A rectangular enclosure is to be built adjacent to a river (no fence is needed along the river side). The other three sides are to be fenced. The area of the enclosure must be 1000 square meters.

If the fencing along the two sides perpendicular to the river costs \$10 per meter and the fencing along the side parallel to the river costs \$5 per meter, what dimensions will minimize the total cost?



Question 6 - Solution

Let: - x = length along the river (no fence needed) - y = depth away from river (fence on both sides)

Area constraint:

$$xy = 1000 \Rightarrow y = \frac{1000}{x}$$

Cost function: - Two sides of length y at \$10/m: cost = $2(10)(y) = 20y$ - One side of length x at \$5/m: cost = $5x$

$$C(x) = 5x + 20 \cdot \frac{1000}{x} = 5x + \frac{20000}{x}$$

Minimize cost — take derivative:

$$C'(x) = 5 - \frac{20000}{x^2}$$

Set derivative to zero:

$$5 = \frac{20000}{x^2} \Rightarrow x^2 = 4000 \Rightarrow x = \sqrt{4000} = 20\sqrt{10}$$

Find y :

$$y = \frac{1000}{x} = \frac{1000}{20\sqrt{10}} = \frac{50}{\sqrt{10}} = 5\sqrt{10}$$

Answer:

$x = 20\sqrt{10} \approx 63.25 \text{ m}$ $y = 5\sqrt{10} \approx 15.81 \text{ m}$

These dimensions minimize the fencing cost.