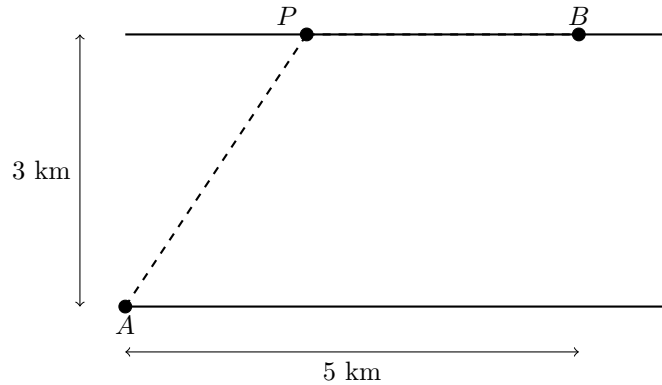


Question 5

A person is standing at point A on the shore of a straight river that is 3 km wide. Point B lies on the opposite shore, 5 km downstream from the point directly opposite A . The person can run at 10 km/h along the shore and swim at 5 km/h directly across the river or at an angle.

The person first swims from A to P , then runs along the opposite shore to B . At what point P between the point directly opposite A and B should the person land to minimize total time to reach B ?



Question 5 - Solution

Let the swimmer land at point P , which is x km from the point directly across from A .

Step 1: Write distances:

Swim distance: $\sqrt{x^2 + 3^2} = \sqrt{x^2 + 9}$, Run distance: $5 - x$

Step 2: Write time expression:

Swimming speed: 5 km/h , Running speed: 10 km/h

$$T(x) = \frac{\sqrt{x^2 + 9}}{5} + \frac{5 - x}{10}$$

Step 3: Minimize time — take derivative:

$$T'(x) = \frac{x}{5\sqrt{x^2 + 9}} - \frac{1}{10}$$

Set $T'(x) = 0$:

$$\frac{x}{5\sqrt{x^2 + 9}} = \frac{1}{10} \Rightarrow \frac{2x}{\sqrt{x^2 + 9}} = 1 \Rightarrow 4x^2 = x^2 + 9 \Rightarrow 3x^2 = 9 \Rightarrow x^2 = 3 \Rightarrow x = \sqrt{3}$$

Step 4: Answer

$$x = \sqrt{3} \approx 1.732 \text{ km}$$

$T_{\min}$ = when the person lands $\sqrt{3}$ km from the point directly across from A .