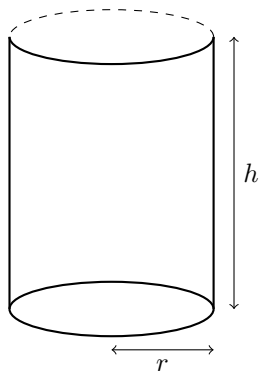


Question 4

A closed right circular cylinder has a fixed volume of $V = 1000 \text{ cm}^3$.

Find the radius and height of the cylinder that minimizes its surface area. Also determine the minimum surface area.



Question 4 - Solution

Let the radius be r , and the height be h . We are given:

$$V = \pi r^2 h = 1000 \Rightarrow h = \frac{1000}{\pi r^2}$$

Surface Area of the Cylinder:

$$A = 2\pi r^2 + 2\pi r h \Rightarrow A(r) = 2\pi r^2 + 2\pi r \cdot \frac{1000}{\pi r^2} = 2\pi r^2 + \frac{2000}{r}$$

Minimize $A(r)$:

Take derivative:

$$A'(r) = 4\pi r - \frac{2000}{r^2}$$

Set $A'(r) = 0$:

$$4\pi r = \frac{2000}{r^2} \Rightarrow 4\pi r^3 = 2000 \Rightarrow r^3 = \frac{500}{\pi} \Rightarrow r = \sqrt[3]{\frac{500}{\pi}}$$

Now compute h :

$$h = \frac{1000}{\pi r^2} = \frac{1000}{\pi \left(\sqrt[3]{\frac{500}{\pi}}\right)^2} = \frac{1000}{\pi} \cdot \left(\frac{\pi}{500}\right)^{2/3} = 1000 \cdot \pi^{-1/3} \cdot 500^{-2/3}$$

Minimum Surface Area:

$$A = 2\pi r^2 + \frac{2000}{r} = 2\pi \left(\sqrt[3]{\frac{500}{\pi}}\right)^2 + \frac{2000}{\sqrt[3]{\frac{500}{\pi}}}$$

$r = \sqrt[3]{\frac{500}{\pi}}$
$h = \frac{1000}{\pi r^2}$
$A_{\min} = 2\pi r^2 + \frac{2000}{r}$