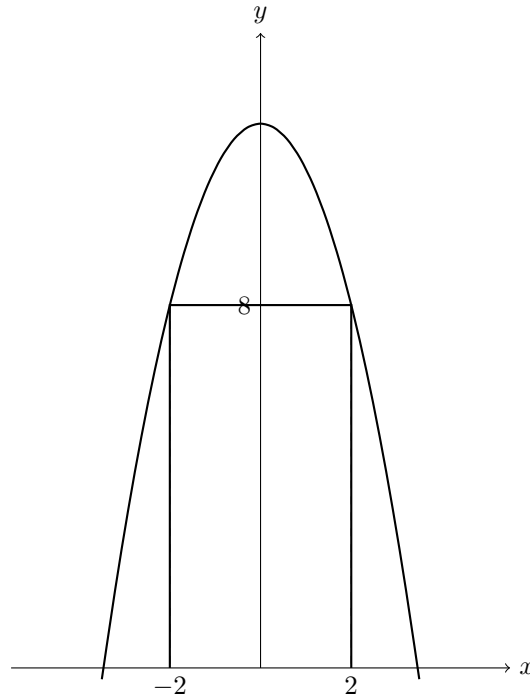


Question 2

A rectangle is inscribed under the parabola $y = 12 - x^2$ such that its base lies on the x -axis and its upper corners lie on the parabola.



Find the dimensions of the rectangle of maximum area.

Question 2 - Solution

Let the top corners of the rectangle be (x, y) and $(-x, y)$, where the rectangle is symmetric about the y -axis.

From the parabola:

$$y = 12 - x^2$$

The width of the rectangle is $2x$, and the height is $y = 12 - x^2$

Area Function:

$$A(x) = \text{width} \times \text{height} = 2x(12 - x^2) = 24x - 2x^3$$

Differentiate:

$$A'(x) = 24 - 6x^2$$

Set derivative to zero:

$$24 - 6x^2 = 0 \Rightarrow x^2 = 4 \Rightarrow x = 2$$

Check Maximum:

Second derivative:

$$A''(x) = -12x \Rightarrow A''(2) = -24 < 0$$

So, $x = 2$ gives a maximum.

Final Dimensions:

$$x = 2 \Rightarrow \text{width} = 2x = 4$$

$$y = 12 - x^2 = 12 - 4 = 8$$

Width = 4
Height = 8
Maximum Area = 32