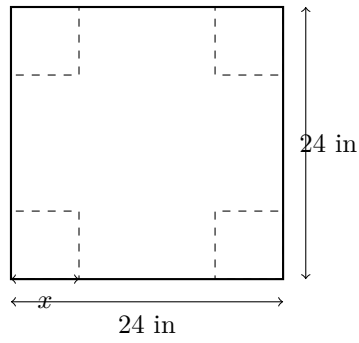


Question 1

A square piece of cardboard 24 inches on each side is to be made into an open-top box by cutting equal squares from each corner and folding up the sides. What size square should be cut from each corner to maximize the volume of the box?

Diagram:



Question 1 - Solution

Let x be the length of the square cut from each corner.

After folding, the dimensions of the box will be:

Height = x

Length and width = $24 - 2x$

Volume:

$$V(x) = x(24 - 2x)^2$$

Step 1: Expand volume function

$$V(x) = x(576 - 96x + 4x^2) = 576x - 96x^2 + 4x^3$$

Step 2: Find critical points

$$V'(x) = 576 - 192x + 12x^2 \Rightarrow 12x^2 - 192x + 576 = 0 \Rightarrow x^2 - 16x + 48 = 0 \Rightarrow x = 8 \pm \sqrt{16} \Rightarrow x = 8 \pm 4$$

$$x = 4 \quad \text{or} \quad x = 12$$

But $x = 12$ makes $24 - 2x = 0 \rightarrow$ not valid

Valid critical point: $x = 4$

Answer: Cut out squares of size 4 in to maximize volume