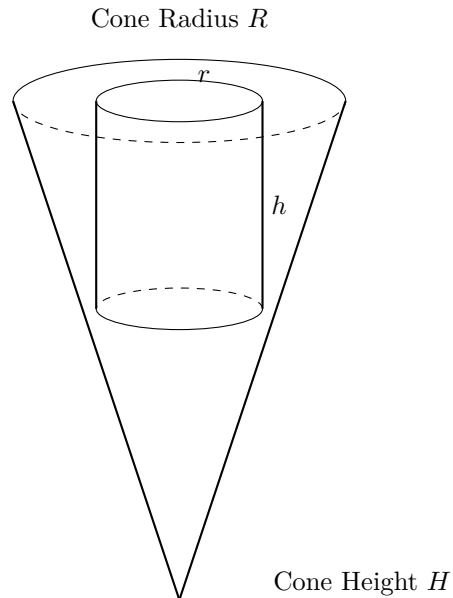


Question 10

A right circular cylinder is inscribed in a right circular cone of height H and base radius R , such that the cylinder shares the same central axis as the cone.

Find the dimensions of the cylinder that maximize its volume.



Question 10 - Solution

Let the height of the cylinder be h , and its base radius be r . We want to maximize:

$$V = \pi r^2 h$$

Step 1: Relationship between r and h

From similar triangles (cone vs cylinder section), we have:

$$\frac{r}{H-h} = \frac{R}{H} \Rightarrow r = \frac{R(H-h)}{H}$$

Substitute into volume:

$$V(h) = \pi \left(\frac{R(H-h)}{H} \right)^2 h = \pi \cdot \frac{R^2(H-h)^2 h}{H^2}$$

Step 2: Maximize $V(h)$

Let's simplify:

$$V(h) = \frac{\pi R^2}{H^2} (H-h)^2 h$$

Let $k = \frac{\pi R^2}{H^2}$, then:

$$V(h) = k(H-h)^2 h$$

Use the product rule:

$$V'(h) = k [2(H-h)(-1) \cdot h + (H-h)^2] = k [-2h(H-h) + (H-h)^2]$$

$$= k(H-h)[(H-h) - 2h] = k(H-h)(H-3h)$$

Set $V'(h) = 0$:

$$(H-h)(H-3h) = 0 \Rightarrow h = H \text{ or } h = \frac{H}{3}$$

But $h = H$ gives 0 volume. So maximum occurs at $\boxed{h = \frac{H}{3}}$

Substitute into r :

$$r = \frac{R(H-H/3)}{H} = \frac{2R}{3}$$

Step 3: Final Answer

Height of cylinder: $h = \frac{H}{3}$
Radius of cylinder: $r = \frac{2R}{3}$
Maximum Volume: $V = \pi r^2 h = \pi \cdot \left(\frac{2R}{3} \right)^2 \cdot \frac{H}{3} = \frac{4\pi R^2 H}{27}$