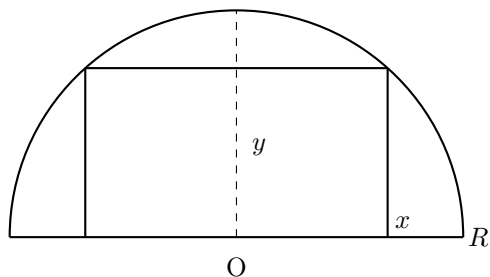


Question 1

A rectangle is inscribed in a semicircle of radius R , with the rectangle's base lying along the diameter of the semicircle.

Find the dimensions of the rectangle that has the maximum area.



Rectangle inscribed in a semicircle

Question 1 - Solution

Let the coordinates of the top-right corner of the rectangle be (x, y) . Since the rectangle is symmetric and lies in a semicircle of radius R centered at the origin:

$$x^2 + y^2 = R^2 \quad (\text{semicircle equation}) \Rightarrow y = \sqrt{R^2 - x^2}$$

Step 1: Area of Rectangle

Since the base is $2x$ and the height is y , area is:

$$A(x) = 2x \cdot y = 2x\sqrt{R^2 - x^2}$$

Step 2: Maximize Area

$$A(x) = 2x\sqrt{R^2 - x^2}$$

Differentiate using the product rule:

$$A'(x) = 2\sqrt{R^2 - x^2} + 2x \cdot \frac{-x}{\sqrt{R^2 - x^2}} = \frac{2(R^2 - x^2) - 2x^2}{\sqrt{R^2 - x^2}} = \frac{2R^2 - 4x^2}{\sqrt{R^2 - x^2}}$$

Set $A'(x) = 0$:

$$2R^2 - 4x^2 = 0 \Rightarrow x^2 = \frac{R^2}{2} \Rightarrow x = \frac{R}{\sqrt{2}}, \quad y = \sqrt{R^2 - x^2} = \frac{R}{\sqrt{2}}$$

Step 3: Max Area and Dimensions

$$\text{Width} = 2x = \sqrt{2}R, \quad \text{Height} = y = \frac{R}{\sqrt{2}}, \quad \text{Area} = 2x \cdot y = R^2$$

Maximum area: $A = R^2$
Width: $2x = \sqrt{2}R$
Height: $y = \frac{R}{\sqrt{2}}$