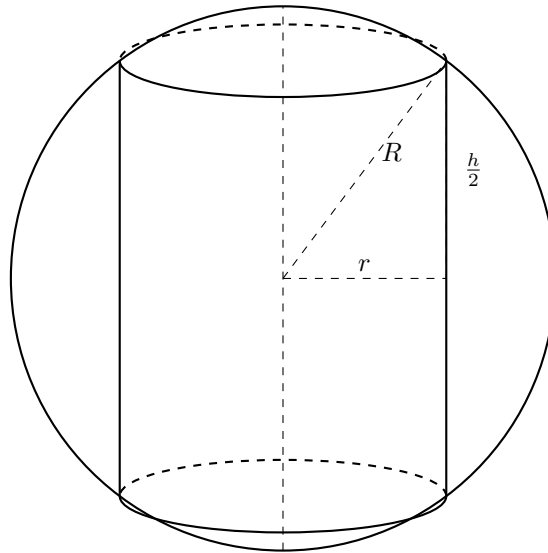


Question 8

A cylinder is inscribed in a sphere of radius R . Find the maximum possible volume of such a cylinder and determine its dimensions.



Cylinder inscribed in a sphere of radius R

Question 8 - Solution

Let the height of the cylinder be h and its radius be r . From the diagram, the diagonal of the right triangle gives the constraint:

$$r^2 + \left(\frac{h}{2}\right)^2 = R^2 \Rightarrow h = 2\sqrt{R^2 - r^2}$$

Step 1: Volume Function

$$V = \pi r^2 h = \pi r^2 \cdot 2\sqrt{R^2 - r^2} = 2\pi r^2 \sqrt{R^2 - r^2}$$

Step 2: Maximize $V(r)$

Let:

$$V(r) = 2\pi r^2 (R^2 - r^2)^{1/2}$$

Differentiate using the product and chain rule:

$$V'(r) = 2\pi \left[2r(R^2 - r^2)^{1/2} - \frac{r^3}{(R^2 - r^2)^{1/2}} \right] = \frac{2\pi}{(R^2 - r^2)^{1/2}} (2r(R^2 - r^2) - r^3)$$

Set $V'(r) = 0$:

$$2r(R^2 - r^2) - r^3 = 0 \Rightarrow r(2R^2 - 3r^2) = 0 \Rightarrow r = 0 \text{ or } r^2 = \frac{2R^2}{3}$$

Step 3: Dimensions

$$r = \sqrt{\frac{2}{3}}R, \quad h = 2\sqrt{R^2 - r^2} = 2\sqrt{R^2 - \frac{2R^2}{3}} = 2\sqrt{\frac{R^2}{3}} = \frac{2R}{\sqrt{3}}$$

Step 4: Max Volume

$$V = \pi r^2 h = \pi \cdot \left(\frac{2R^2}{3}\right) \cdot \frac{2R}{\sqrt{3}} = \frac{4\pi R^3}{3\sqrt{3}}$$

Radius: $r = \sqrt{\frac{2}{3}}R$
Height: $h = \frac{2R}{\sqrt{3}}$
Max Volume: $V = \frac{4\pi R^3}{3\sqrt{3}}$