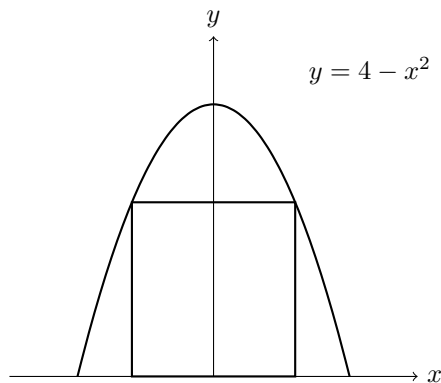


Question 7

Find the dimensions of the rectangle of maximum area that can be inscribed under the parabola

$$y = 4 - x^2$$

with its base lying along the x-axis and the top two corners touching the curve.



Question 7 - Solution

Let the rectangle extend from $-x$ to x , so its width is $2x$, and its height is the y-value of the curve:

$$\text{Height} = y = 4 - x^2$$

Step 1: Area Function

$$A(x) = \text{width} \times \text{height} = 2x(4 - x^2) = 8x - 2x^3$$

Step 2: Maximize the Area

Differentiate:

$$A'(x) = 8 - 6x^2$$

Set $A'(x) = 0$:

$$8 - 6x^2 = 0 \Rightarrow x^2 = \frac{4}{3} \Rightarrow x = \frac{2}{\sqrt{3}} \approx 1.1547$$

Step 3: Dimensions of the Rectangle

$$\text{Width} = 2x = \frac{4}{\sqrt{3}} \approx 2.309$$

$$\text{Height} = 4 - x^2 = 4 - \frac{4}{3} = \frac{8}{3} \approx 2.667$$

Maximum Area Rectangle Dimensions: Width ≈ 2.309 , Height ≈ 2.667

Step 4: Confirm Maximum

$$A''(x) = -12x < 0 \quad \text{at } x > 0 \Rightarrow \text{maximum confirmed.}$$