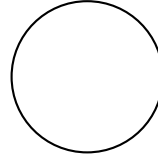


Question 6

A 100 cm long wire is to be cut into two pieces. One piece is bent to form a square, and the other is bent to form a circle. How should the wire be cut to minimize the total enclosed area?



Square



Circle

Question 6 - Solution

Let:

x = length used for the square, $0 < x < 100 \Rightarrow 100 - x$ = length used for the circle

Step 1: Area Expressions

Square:

$$\text{Perimeter} = x \Rightarrow \text{Side} = \frac{x}{4}, \quad A_{\text{square}} = \left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$$

Circle:

$$\text{Circumference} = 100 - x = 2\pi r \Rightarrow r = \frac{100 - x}{2\pi} \Rightarrow A_{\text{circle}} = \pi r^2 = \pi \left(\frac{100 - x}{2\pi}\right)^2 = \frac{(100 - x)^2}{4\pi}$$

Total Area:

$$A(x) = \frac{x^2}{16} + \frac{(100 - x)^2}{4\pi}$$

Step 2: Minimize Area

Differentiate:

$$A'(x) = \frac{1}{8}x - \frac{1}{2\pi}(100 - x)$$

Set $A'(x) = 0$:

$$\frac{1}{8}x = \frac{1}{2\pi}(100 - x) \Rightarrow \pi x = 4(100 - x) \Rightarrow \pi x = 400 - 4x \Rightarrow x(\pi + 4) = 400 \Rightarrow x = \frac{400}{\pi + 4}$$

Step 3: Approximate and Justify Minimum

$$x \approx \frac{400}{3.1416 + 4} = \frac{400}{7.1416} \approx 56.0$$

$x \approx 56.0$ cm for the square	44.0 cm for the circle
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Second derivative:

$$A''(x) = \frac{1}{8} + \frac{1}{2\pi} > 0 \Rightarrow \text{This critical point is a minimum.}$$

This wire division minimizes the total area.