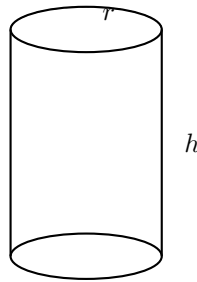


Question 3**Problem:**

A cylindrical can is to be made to hold a volume of 1000 cm^3 . The material for the top and bottom costs twice as much per square centimeter as the material for the side.

- (a) Find the dimensions (radius and height) of the can that minimize the cost of the material.
- (b) What is the minimum total cost in terms of material area?

Diagram:

Question 3 - Solution

Let: - r : radius of the cylinder - h : height of the cylinder

Volume constraint:

$$V = \pi r^2 h = 1000 \quad \Rightarrow \quad h = \frac{1000}{\pi r^2}$$

Cost: - Side area = $2\pi r h$, unit cost = 1 - Top and bottom area = $2\pi r^2$, unit cost = 2

Total cost C (proportional to material area):

$$C = 2\pi r h + 2 \cdot 2\pi r^2 = 2\pi r h + 4\pi r^2$$

Substitute h :

$$C(r) = 2\pi r \cdot \frac{1000}{\pi r^2} + 4\pi r^2 = \frac{2000}{r} + 4\pi r^2$$

Differentiate:

$$C'(r) = -\frac{2000}{r^2} + 8\pi r$$

Set derivative to 0:

$$-\frac{2000}{r^2} + 8\pi r = 0 \Rightarrow 8\pi r^3 = 2000 \Rightarrow r^3 = \frac{250}{\pi}$$

$$r = \sqrt[3]{\frac{250}{\pi}}, \quad h = \frac{1000}{\pi r^2}$$

(a) Optimal Dimensions:

$$\boxed{r = \sqrt[3]{\frac{250}{\pi}}, \quad h = \frac{1000}{\pi r^2}}$$

(b) Minimum Cost (in material area units):

$$\boxed{C = \frac{2000}{r} + 4\pi r^2} \quad \text{where } r = \sqrt[3]{\frac{250}{\pi}}$$