

Question 10**Problem:**

Let

$$f(x) = \sin(x)$$

on the interval $[0, \pi]$.

- (a) Verify that $f(x)$ satisfies the conditions of the Mean Value Theorem on $[0, \pi]$.
- (b) Find all values of $c \in (0, \pi)$ such that

$$f'(c) = \frac{f(\pi) - f(0)}{\pi - 0}.$$

Question 10 - Solution

We are given:

$$f(x) = \sin(x)$$

(a) **Conditions for MVT:**

- $f(x)$ is continuous on $[0, \pi]$ - $f(x)$ is differentiable on $(0, \pi)$
MVT applies.

(b) **Apply MVT:**

$$\begin{aligned} f(\pi) &= 0, & f(0) &= 0 \\ \frac{f(\pi) - f(0)}{\pi - 0} &= \frac{0 - 0}{\pi} = 0 \end{aligned}$$

We need:

$$f'(c) = \cos(c) = 0 \Rightarrow c = \frac{\pi}{2}$$

$$\boxed{c = \frac{\pi}{2}}$$

Graph of $f(x) = \sin(x)$ on $[0, \pi]$:

