

Question 9**Problem:**

Let

$$f(x) = \frac{1}{x}$$

on the interval $[1, 3]$.

- (a) Verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem.
- (b) Find all numbers $c \in (1, 3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3 - 1}.$$

Question 9 - Solution

We are given:

$$f(x) = \frac{1}{x}$$

(a) **Conditions for MVT:**

- $f(x)$ is continuous on $[1, 3]$
 - $f(x)$ is differentiable on $(1, 3)$
- Therefore, MVT applies.

(b) **Apply MVT:**

$$f(3) = \frac{1}{3}, \quad f(1) = 1$$

$$\frac{f(3) - f(1)}{3 - 1} = \frac{\frac{1}{3} - 1}{2} = \frac{-\frac{2}{3}}{2} = -\frac{1}{3}$$

Now compute derivative:

$$f'(x) = \frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$$

$$f'(c) = -\frac{1}{c^2} = -\frac{1}{3} \Rightarrow c^2 = 3 \Rightarrow c = \sqrt{3}$$

$$c = \sqrt{3} \in (1, 3)$$

Graph of $f(x) = \frac{1}{x}$ on $[1, 3]$:

