

Question 8**Problem:**

Let

$$f(x) = \sin(x)$$

on the interval $[0, \pi]$.

- (a) Verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem.
- (b) Find all numbers $c \in (0, \pi)$ such that

$$f'(c) = \frac{f(\pi) - f(0)}{\pi - 0}.$$

Question 8 - Solution

We are given:

$$f(x) = \sin(x)$$

(a) MVT Conditions:

- $f(x)$ is continuous on $[0, \pi]$: Sine is continuous everywhere.
 - $f(x)$ is differentiable on $(0, \pi)$: Sine is differentiable everywhere.
- MVT applies.

(b) Apply MVT:

Average rate of change:

$$f(\pi) = \sin(\pi) = 0, \quad f(0) = \sin(0) = 0$$

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{0 - 0}{\pi} = 0$$

We want:

$$f'(c) = \cos(c) = 0 \Rightarrow c = \frac{\pi}{2}$$

$$c = \frac{\pi}{2}$$

Graph of $f(x) = \sin(x)$ on $[0, \pi]$:

