

Question 7**Problem:**

Let

$$f(x) = \frac{x}{x+1}$$

on the interval $[1, 4]$.

- (a) Show that the hypotheses of the Mean Value Theorem are satisfied.
- (b) Find all values of $c \in (1, 4)$ such that

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}.$$

Question 7 - Solution

We are given:

$$f(x) = \frac{x}{x+1}$$

(a) **MVT Conditions:**

- $f(x)$ is continuous on $[1, 4]$: The function is a rational function with no discontinuities on this interval.
- $f(x)$ is differentiable on $(1, 4)$: Differentiable as a quotient of differentiable functions, and denominator never zero.

MVT applies.

(b) **Compute the average rate of change:**

$$\begin{aligned} f(1) &= \frac{1}{2}, & f(4) &= \frac{4}{5} \\ \frac{f(4) - f(1)}{4 - 1} &= \frac{\frac{4}{5} - \frac{1}{2}}{3} = \frac{\frac{8-5}{10}}{3} = \frac{3}{10} \cdot \frac{1}{3} = \frac{1}{10} \end{aligned}$$

Find derivative:

$$f'(x) = \frac{(x+1)(1) - x(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$$

Set:

$$\frac{1}{(c+1)^2} = \frac{1}{10} \Rightarrow (c+1)^2 = 10 \Rightarrow c+1 = \sqrt{10} \Rightarrow c = \sqrt{10} - 1$$

Since $\sqrt{10} \approx 3.16$, we find $c \approx 2.16 \in (1, 4)$

$c = \sqrt{10} - 1$

Graph of $f(x)$ and Secant Line:

