

Question 6**Problem:**

Let

$$f(x) = \ln(x^2 + 1)$$

on the interval $[-1, 2]$.

- (a) Verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem on the interval.
- (b) Find all values of $c \in (-1, 2)$ such that

$$f'(c) = \frac{f(2) - f(-1)}{2 - (-1)}.$$

Question 6 - Solution

The function $f(x) = \ln(1 + x^2)$ is continuous on $[-1, 2]$ and differentiable on $(-1, 2)$, so the Mean Value Theorem applies.

$$\frac{f(2) - f(-1)}{2 - (-1)} = \frac{\ln(5/2)}{3}, \quad f'(c) = \frac{2c}{1 + c^2}.$$

Let $L = \ln(5/2)$. Then $Lc^2 - 6c + L = 0$, so

$$c = \frac{3 \pm \sqrt{9 - L^2}}{L}.$$

The plus root exceeds 2. The only admissible value is

$$c = \frac{3 - \sqrt{9 - [\ln(5/2)]^2}}{\ln(5/2)} \approx 0.156453.$$