

Question 5**Problem:**

Let

$$f(x) = \frac{x}{x+2} \quad \text{on the interval } [1, 4].$$

- (a) Verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem on $[1, 4]$.
- (b) Find all values $c \in (1, 4)$ such that

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}.$$

Question 5 - Solution

We are given:

$$f(x) = \frac{x}{x+2}$$

(a) **Check MVT conditions:**

- $f(x)$ is a rational function with denominator $x+2 \neq 0$ on $[1, 4]$ continuous on $[1, 4]$. - It is differentiable on $(1, 4)$ as it's smooth where defined.

MVT applies.

(b) **Compute average rate of change:**

$$f(1) = \frac{1}{3}, \quad f(4) = \frac{4}{6} = \frac{2}{3}$$

$$\frac{f(4) - f(1)}{4 - 1} = \frac{\frac{2}{3} - \frac{1}{3}}{3} = \frac{1}{9}$$

Find derivative:

Use quotient rule:

$$f'(x) = \frac{(x+2)(1) - x(1)}{(x+2)^2} = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2}$$

Set:

$$\frac{2}{(c+2)^2} = \frac{1}{9} \Rightarrow (c+2)^2 = 18 \Rightarrow c+2 = \pm\sqrt{18} = \pm 3\sqrt{2} \Rightarrow c = -2 \pm 3\sqrt{2}$$

Now, only one of these is in the interval $(1, 4)$:

$$c = -2 + 3\sqrt{2} \approx -2 + 4.24 = 2.24$$

$$\boxed{c = -2 + 3\sqrt{2}} \text{ is in } (1, 4)$$

Graph of $f(x)$ and secant line:

