

Question 2**Problem:**

Let

$$f(x) = x^3 - 3x^2 + 2x, \quad \text{on the interval } [1, 3]$$

- (a) Verify that f satisfies the hypotheses of the Mean Value Theorem on $[1, 3]$.
- (b) Find all values of $c \in (1, 3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3 - 1}$$

Question 2 - Solution

We are given:

$$f(x) = x^3 - 3x^2 + 2x$$

(a) Hypotheses of the MVT:

- $f(x)$ is a polynomial continuous on $[1, 3]$ - $f(x)$ is differentiable on $(1, 3)$
Therefore, MVT applies.

(b) Apply the MVT:

Compute:

$$f(3) = 27 - 27 + 6 = 6, \quad f(1) = 1 - 3 + 2 = 0$$

$$\text{Average rate of change} = \frac{6 - 0}{3 - 1} = \frac{6}{2} = 3$$

Compute derivative:

$$f'(x) = 3x^2 - 6x + 2$$

Set $f'(c) = 3$:

$$3c^2 - 6c + 2 = 3 \quad \Rightarrow \quad 3c^2 - 6c - 1 = 0$$

Solve:

$$c = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(-1)}}{2(3)} = \frac{6 \pm \sqrt{36 + 12}}{6} = \frac{6 \pm \sqrt{48}}{6} = \frac{6 \pm 4\sqrt{3}}{6} = \frac{3 \pm 2\sqrt{3}}{3}$$

Answer:

$$\boxed{c = \frac{3 \pm 2\sqrt{3}}{3}} \quad \text{Only the value(s) in } (1, 3) \text{ are valid.}$$

Since:

$$\frac{3 - 2\sqrt{3}}{3} \approx 0.845 < 1 \quad (\text{not valid})$$

$$\frac{3 + 2\sqrt{3}}{3} \approx 2.155 \in (1, 3)$$

Final answer:

$$\boxed{c = \frac{3 + 2\sqrt{3}}{3}}$$

Graph of $f(x) = x^3 - 3x^2 + 2x$ on $[1, 3]$:

