

Question 2**Problem:**

Let

$$f(x) = \ln(x), \quad \text{for } x \in [1, 4]$$

- (a) Verify that f satisfies the hypotheses of the Mean Value Theorem on the interval.
- (b) Find all values of $c \in (1, 4)$ such that

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}$$

Question 2 - Solution

We are given:

$$f(x) = \ln(x)$$

(a) **Check MVT hypotheses:**

1. $f(x)$ is continuous on $[1, 4]$: Yes, $\ln(x)$ is continuous for $x > 0$, so continuous on $[1, 4]$
 2. $f(x)$ is differentiable on $(1, 4)$: Yes, $\ln(x)$ is differentiable for $x > 0$, so differentiable on $(1, 4)$
- The Mean Value Theorem applies.

(b) **Find the value(s) of c :**

Compute average rate of change:

$$\frac{f(4) - f(1)}{4 - 1} = \frac{\ln(4) - \ln(1)}{3} = \frac{\ln(4)}{3} = \frac{2\ln(2)}{3}$$

Now compute $f'(x)$:

$$f'(x) = \frac{1}{x}$$

Set:

$$f'(c) = \frac{1}{c} = \frac{2\ln(2)}{3} \Rightarrow c = \frac{3}{2\ln(2)}$$

Answer:

$$c = \frac{3}{2\ln(2)} \approx 2.164$$

Graph of $f(x) = \ln(x)$ on $[1, 4]$:

