

Question 1

Let

$$f(x) = x^2 + 2x$$

- (a) Verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem on the interval $[1, 4]$.
- (b) Find all numbers $c \in (1, 4)$ that satisfy the conclusion of the Mean Value Theorem.

Question 1 - Solution

We are given

$$f(x) = x^2 + 2x$$

(a) Verifying the hypotheses

The function $f(x)$ is a polynomial, so it is continuous on the closed interval $[1, 4]$ and differentiable on the open interval $(1, 4)$.

Therefore, the hypotheses of the Mean Value Theorem are satisfied.

(b) Applying the Mean Value Theorem

First compute the derivative:

$$f'(x) = 2x + 2$$

Next compute the average rate of change of f on $[1, 4]$:

$$\frac{f(4) - f(1)}{4 - 1} = \frac{(16 + 8) - (1 + 2)}{3} = \frac{21}{3} = 7$$

Set the derivative equal to this value:

$$2c + 2 = 7 \Rightarrow 2c = 5 \Rightarrow c = \frac{5}{2}$$

Thus, the number that satisfies the conclusion of the Mean Value Theorem is

$$c = \frac{5}{2}$$

Graph of $f(x)$, the secant line, and the tangent line at c

