

Question 9

Let

$$f(x) = x^3 - 3x$$

- (a) Determine the intervals where the graph of f is concave up and concave down.
- (b) Find all inflection points.

Question 9 - Solution

We are given

$$f(x) = x^3 - 3x$$

First derivative

$$f'(x) = 3x^2 - 3$$

Second derivative

$$f''(x) = 6x$$

(a) Concavity

Setting $f''(x) = 0$ gives

$$x = 0.$$

For $x < 0$, the second derivative is negative, so the graph is concave down.

For $x > 0$, the second derivative is positive, so the graph is concave up.

Concave down on

$$(-\infty, 0)$$

Concave up on

$$(0, \infty)$$

(b) Inflection point

Because the concavity changes at $x = 0$, an inflection point occurs there.

$$f(0) = 0$$

Inflection point at

$$(0, 0)$$

Graph of $f(x)$

