

Question 7

Let

$$f(x) = x^3 - 6x^2 + 12x - 5$$

- (a) Determine the intervals where the graph of f is concave up and concave down.
- (b) Find all inflection points.

Question 7 - Solution

We are given

$$f(x) = x^3 - 6x^2 + 12x - 5$$

First derivative

$$f'(x) = 3x^2 - 12x + 12$$

Second derivative

$$f''(x) = 6x - 12$$

(a) Concavity

Setting $f''(x) = 0$ gives

$$x = 2.$$

For $x < 2$, the second derivative is negative, so the graph is concave down.

For $x > 2$, the second derivative is positive, so the graph is concave up.

Concave down on

$$(-\infty, 2)$$

Concave up on

$$(2, \infty)$$

(b) Inflection point

Because the concavity changes at $x = 2$, an inflection point occurs there.

$$f(2) = 3$$

Inflection point at

$$(2, 3)$$

Graph of $f(x)$

