

Question 2

Let

$$f(x) = x^4 - 4x^3$$

- (a) Find the intervals where the graph of f is concave up and concave down.
- (b) Identify any inflection points.

Question 2 - Solution

We are given

$$f(x) = x^4 - 4x^3$$

First derivative

$$f'(x) = 4x^3 - 12x^2$$

Second derivative

$$f''(x) = 12x^2 - 24x = 12x(x - 2)$$

(a) Concavity

Setting $f''(x) = 0$ gives the critical values

$$x = 0 \quad \text{and} \quad x = 2.$$

For $x < 0$, the second derivative is positive, so the graph is concave up. For $0 < x < 2$, the second derivative is negative, so the graph is concave down. For $x > 2$, the second derivative is positive, so the graph is concave up.

Concave up on

$$(-\infty, 0) \cup (2, \infty)$$

Concave down on

$$(0, 2)$$

(b) Inflection points

Because the concavity changes at both $x = 0$ and $x = 2$, inflection points occur at these values.

$$f(0) = 0 \quad f(2) = 16 - 32 = -16$$

Inflection points at

$$(0, 0) \quad \text{and} \quad (2, -16)$$

Graph of $f(x)$

