

Question 1

Let

$$f(x) = x^3 - 3x^2 - 9x + 5$$

- (a) Find the intervals where $f(x)$ is increasing or decreasing.
- (b) Identify all local maxima and minima.
- (c) Determine intervals of concavity and any inflection points.

Question 1 - Solution

We are given

$$f(x) = x^3 - 3x^2 - 9x + 5$$

(a) First derivative

$$f'(x) = 3x^2 - 6x - 9 = 3(x - 3)(x + 1)$$

The critical points are $x = -1$ and $x = 3$.

For $x < -1$, the derivative is positive. For $-1 < x < 3$, the derivative is negative. For $x > 3$, the derivative is positive.

Therefore, the function is increasing on

$$(-\infty, -1) \cup (3, \infty)$$

and decreasing on

$$(-1, 3).$$

(b) Local extrema

At $x = -1$, the function changes from increasing to decreasing, so there is a local maximum.

$$f(-1) = 10$$

At $x = 3$, the function changes from decreasing to increasing, so there is a local minimum.

$$f(3) = -22$$

Local maximum at $(-1, 10)$

Local minimum at $(3, -22)$

(c) Second derivative

$$f''(x) = 6x - 6$$

Setting $f''(x) = 0$ gives $x = 1$.

For $x < 1$, the second derivative is negative, so the graph is concave down. For $x > 1$, the second derivative is positive, so the graph is concave up.

The inflection point occurs at

$$f(1) = -6$$

Concave down on $(-\infty, 1)$

Concave up on $(1, \infty)$

Inflection point at $(1, -6)$

Graph of $f(x)$

