

Question 10**Problem:**

Let

$$f(x) = \frac{x}{x^2 + 1}$$

- (a) Find the critical points of $f(x)$.
- (b) Determine where the function is increasing and decreasing.
- (c) Identify and classify any local extrema.

Question 10 - Solution

We are given:

$$f(x) = \frac{x}{x^2 + 1}$$

(a) Critical Points:

Use the quotient rule:

$$f'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{-x^2 + 1}{(x^2 + 1)^2}$$

Set numerator equal to zero:

$$-x^2 + 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Critical points: $x = -1, x = 1$

(b) Increasing/Decreasing Intervals:

Analyze the sign of $f'(x) = \frac{1-x^2}{(x^2+1)^2}$

Denominator is always positive. So sign depends on numerator $1 - x^2$:

$x < -1$: $f'(x) < 0$ $-1 < x < 1$: $f'(x) > 0$ $x > 1$: $f'(x) < 0$

Increasing: $(-1, 1)$ **Decreasing:** $(-\infty, -1) \cup (1, \infty)$

(c) Local Extrema:

$f'(x)$ changes from negative to positive at $x = -1 \rightarrow$ local min $f(-1) = \frac{-1}{1+1} = -\frac{1}{2}$

$f'(x)$ changes from positive to negative at $x = 1 \rightarrow$ local max $f(1) = \frac{1}{1+1} = \frac{1}{2}$

Conclusion: Local minimum at $(-1, -\frac{1}{2})$, Local maximum at $(1, \frac{1}{2})$

Graph of $f(x) = \frac{x}{x^2+1}$:

