

Question 9**Problem:**

Let

$$f(x) = x^3 - 6x^2 + 9x + 1$$

- (a) Find all critical points of $f(x)$.
- (b) Determine the intervals where $f(x)$ is increasing or decreasing.
- (c) Identify and classify all local extrema.

Question 9 - Solution

We are given:

$$f(x) = x^3 - 6x^2 + 9x + 1$$

(a) Critical Points:

Compute the first derivative:

$$f'(x) = 3x^2 - 12x + 9$$

Set derivative equal to 0:

$$3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow x = 1, x = 3$$

Critical points: $x = 1, x = 3$

(b) Increasing/Decreasing Intervals:

Test sign of $f'(x)$ in each interval:

$(-\infty, 1)$: pick $x = 0$, $f'(0) = 9 > 0$

$(1, 3)$: pick $x = 2$, $f'(2) = 3(4) - 12(2) + 9 = 12 - 24 + 9 = -3 < 0$

$(3, \infty)$: pick $x = 4$, $f'(4) = 3(16) - 48 + 9 = 48 - 48 + 9 = 9 > 0$

Increasing: $(-\infty, 1) \cup (3, \infty)$ **Decreasing:** $(1, 3)$

(c) Local Extrema:

At $x = 1$: increasing to decreasing $\rightarrow$ local max $f(1) = 1 - 6 + 9 + 1 = 5$

At $x = 3$: decreasing to increasing $\rightarrow$ local min $f(3) = 27 - 54 + 27 + 1 = 1$

Conclusion:

Local maximum at $(1, 5)$, Local minimum at $(3, 1)$

Graph of $f(x) = x^3 - 6x^2 + 9x + 1$:

