

Question 8**Problem:**

Let

$$f(x) = \frac{x^2}{x+2}$$

- (a) Find the domain of $f(x)$ and determine all critical points.
- (b) Identify intervals where $f(x)$ is increasing or decreasing.
- (c) Classify each critical point as a local maximum, local minimum, or neither.

Question 8 - Solution

We are given:

$$f(x) = \frac{x^2}{x+2}$$

(a) Domain and Critical Points:

The function is undefined at $x = -2$, so the domain is:

$$(-\infty, -2) \cup (-2, \infty)$$

Find the derivative using the quotient rule:

$$f'(x) = \frac{(2x)(x+2) - x^2(1)}{(x+2)^2} = \frac{2x(x+2) - x^2}{(x+2)^2} = \frac{2x^2 + 4x - x^2}{(x+2)^2} = \frac{x^2 + 4x}{(x+2)^2}$$

Set the derivative to zero:

$$f'(x) = 0 \Rightarrow x^2 + 4x = 0 \Rightarrow x(x+4) = 0 \Rightarrow x = 0, x = -4$$

Critical Points: $x = -4$ and $x = 0$

(b) Test Intervals for Increasing/Decreasing:

Test sign of $f'(x) = \frac{x(x+4)}{(x+2)^2}$

Interval $(-\infty, -4)$: pick $x = -5$: $f'(-5) = \frac{-5(-1)}{(-3)^2} = \frac{5}{9} > 0$

Interval $(-4, -2)$: pick $x = -3$: $f'(-3) = \frac{-3(1)}{(1)^2} = -3 < 0$

Interval $(-2, 0)$: pick $x = -1$: $f'(-1) = \frac{-1(3)}{(1)^2} = -3 < 0$

Interval $(0, \infty)$: pick $x = 1$: $f'(1) = \frac{1(5)}{(3)^2} = \frac{5}{9} > 0$

Increasing: $(-\infty, -4) \cup (0, \infty)$ **Decreasing:** $(-4, -2) \cup (-2, 0)$

(c) Classify Critical Points:

At $x = -4$: changes from increasing to decreasing $\rightarrow$ local max $f(-4) = \frac{16}{-2} = -8$

At $x = 0$: changes from decreasing to increasing $\rightarrow$ local min $f(0) = 0$

Conclusion:

Local maximum at $(-4, -8)$, Local minimum at $(0, 0)$

Graph of $f(x) = \frac{x^2}{x+2}$:

