

Question 7**Problem:**

Let

$$f(x) = x^3 - 3x^2 - 9x + 5$$

- (a) Find the critical points of $f(x)$ and determine the intervals where $f(x)$ is increasing or decreasing.
- (b) Identify all local maximum and minimum values.

Question 7 - Solution

We are given:

$$f(x) = x^3 - 3x^2 - 9x + 5$$

(a) **First Derivative:**

$$f'(x) = 3x^2 - 6x - 9$$

Set $f'(x) = 0$ to find critical points:

$$3x^2 - 6x - 9 = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0 \Rightarrow x = -1, 3$$

Test Intervals:

$(-\infty, -1)$: pick $x = -2$: $f'(-2) = 3(-2)^2 - 6(-2) - 9 = 12 + 12 - 9 = 15 > 0$

$(-1, 3)$: pick $x = 0$: $f'(0) = -9 < 0$

$(3, \infty)$: pick $x = 4$: $f'(4) = 3(16) - 6(4) - 9 = 48 - 24 - 9 = 15 > 0$

Conclusion:

$f(x)$ is increasing on $(-\infty, -1) \cup (3, \infty)$

$f(x)$ is decreasing on $(-1, 3)$

(b) **Local Extrema:**

At $x = -1$: changes from increasing to decreasing $\rightarrow$ local maximum

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 5 = -1 - 3 + 9 + 5 = 10$$

At $x = 3$: changes from decreasing to increasing $\rightarrow$ local minimum

$$f(3) = 27 - 27 - 27 + 5 = -22$$

Answer:

Local maximum at $(-1, 10)$, Local minimum at $(3, -22)$

Graph of $f(x) = x^3 - 3x^2 - 9x + 5$:

