

Question 6**Problem:**

Let

$$f(x) = \frac{x^2}{x+1}$$

- (a) Find the critical points of $f(x)$ and determine where $f(x)$ is increasing or decreasing.
- (b) Identify any local maximum or minimum values.

Question 6 - Solution

We are given:

$$f(x) = \frac{x^2}{x+1}$$

(a) First Derivative:

Use the quotient rule:

$$f'(x) = \frac{(2x)(x+1) - x^2(1)}{(x+1)^2} = \frac{2x(x+1) - x^2}{(x+1)^2} = \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$$

Set $f'(x) = 0$:

$$\frac{x(x+2)}{(x+1)^2} = 0 \Rightarrow x = 0, -2$$

Also, note $f(x)$ is undefined at $x = -1$

Test intervals:

$(-\infty, -2)$: pick $x = -3$: numerator is positive, denominator is positive $\rightarrow f' > 0$

$(-2, -1)$: pick $x = -1.5$: numerator is negative, denominator is positive $\rightarrow f' < 0$

$(-1, 0)$: pick $x = -0.5$: numerator is negative, denominator is positive $\rightarrow f' < 0$

$(0, \infty)$: pick $x = 1$: numerator is positive, denominator is positive $\rightarrow f' > 0$

Conclusion:

$f(x)$ increases on $(-\infty, -2) \cup (0, \infty)$

$f(x)$ decreases on $(-2, -1) \cup (-1, 0)$

(b) Local Extrema:

At $x = -2$: from increasing to decreasing $\rightarrow$ local max $f(-2) = \frac{(-2)^2}{-2+1} = \frac{4}{-1} = -4$

At $x = 0$: from decreasing to increasing $\rightarrow$ local min $f(0) = 0$

Answer:

Local maximum at $(-2, -4)$, Local minimum at $(0, 0)$

Graph of $f(x) = \frac{x^2}{x+1}$:

