

Question 5**Problem:**

Let

$$f(x) = x^4 - 4x^3 + 6$$

- (a) Find the intervals where $f(x)$ is increasing or decreasing.
- (b) Identify any local maximum and minimum values.

Question 5 - Solution

We are given:

$$f(x) = x^4 - 4x^3 + 6$$

(a) **First Derivative:**

$$f'(x) = 4x^3 - 12x^2$$

$$f'(x) = 4x^2(x - 3)$$

Set $f'(x) = 0$:

$$4x^2(x - 3) = 0 \Rightarrow x = 0, 3$$

Test intervals:

For $x < 0$: pick $x = -1$: $f'(-1) = 4(-1)^2(-1 - 3) = 4(1)(-4) = -16 < 0$

For $0 < x < 3$: pick $x = 1$: $f'(1) = 4(1)^2(1 - 3) = 4(-2) = -8 < 0$

For $x > 3$: pick $x = 4$: $f'(4) = 4(16)(1) = 64 > 0$

Conclusion:

$f(x)$ is decreasing on $(-\infty, 3)$

$f(x)$ is increasing on $(3, \infty)$

(b) **Local Extrema:**

$f'(x)$ does not change sign around $x = 0$ (negative on both sides) $\rightarrow$ no extremum at $x = 0$ At $x = 3$:
from decreasing to increasing $\rightarrow$ local minimum

Evaluate:

$$f(3) = 3^4 - 4(3)^3 + 6 = 81 - 108 + 6 = -21$$

Local Minimum: $(3, -21)$

Graph of $f(x) = x^4 - 4x^3 + 6$:

