

Question 3**Problem**

Consider the function

$$f(x) = \frac{x^2 - 4x}{x + 1}$$

- (a) Find the intervals where $f(x)$ is increasing or decreasing.
- (b) Identify any local maximum or minimum points.

Question 3 - Solution

We are given

$$f(x) = \frac{x^2 - 4x}{x + 1} = \frac{x(x - 4)}{x + 1}, \quad x \neq -1$$

(a) First derivative

Using the quotient rule,

$$f'(x) = \frac{(x + 1)(2x - 4) - (x^2 - 4x)}{(x + 1)^2} = \frac{x^2 + 2x - 4}{(x + 1)^2}.$$

Setting the numerator equal to zero gives

$$x^2 + 2x - 4 = 0 \Rightarrow x = -1 \pm \sqrt{5}.$$

Since $(x + 1)^2 > 0$ for $x \neq -1$, the sign of $f'(x)$ is determined by the numerator.

Answer

The function is increasing on

$$(-\infty, -1 - \sqrt{5}) \cup (-1 + \sqrt{5}, \infty).$$

The function is decreasing on

$$(-1 - \sqrt{5}, -1) \cup (-1, -1 + \sqrt{5}).$$

(b) Local extrema

At $x = -1 - \sqrt{5}$, the derivative changes from positive to negative, so there is a local maximum.

At $x = -1 + \sqrt{5}$, the derivative changes from negative to positive, so there is a local minimum.

$$f(-1 - \sqrt{5}) = -6 - 2\sqrt{5}, \quad f(-1 + \sqrt{5}) = -6 + 2\sqrt{5}.$$

Answer

Local maximum at

$$(-1 - \sqrt{5}, -6 - 2\sqrt{5})$$

Local minimum at

$$(-1 + \sqrt{5}, -6 + 2\sqrt{5})$$

Graph of $f(x)$ 