

Question 2**Problem**

Given the function

$$f(x) = x^3 - 3x^2 - 9x + 5$$

- (a) Find the intervals where the function is increasing and decreasing.
- (b) Identify and classify all local extrema.

Question 2 - Solution

We are given

$$f(x) = x^3 - 3x^2 - 9x + 5$$

(a) **First derivative**

$$f'(x) = 3x^2 - 6x - 9 = 3(x - 3)(x + 1)$$

The critical points are

$$x = -1 \quad \text{and} \quad x = 3$$

We test the sign of $f'(x)$.

For $x < -1$, $f'(x) > 0$. For $-1 < x < 3$, $f'(x) < 0$. For $x > 3$, $f'(x) > 0$.

Therefore, the function is increasing on $(-\infty, -1) \cup (3, \infty)$

and decreasing on $(-1, 3)$.

(b) **Local extrema**

Since the function changes from increasing to decreasing at $x = -1$, there is a local maximum:

$$f(-1) = 10$$

Since the function changes from decreasing to increasing at $x = 3$, there is a local minimum:

$$f(3) = -22$$

Answer

Local maximum at $(-1, 10)$

Local minimum at $(3, -22)$

Graph of $f(x)$

