

Question 1

Let:

$$f(x) = x^4 - 4x^3 + 6x^2$$

- (a) Find the intervals where $f(x)$ is concave up or concave down.
- (b) Determine the inflection points of $f(x)$, if any.

Question 1 - Solution

We are given:

$$f(x) = x^4 - 4x^3 + 6x^2$$

Step 1: Find the second derivative

$$f'(x) = 4x^3 - 12x^2 + 12x$$

$$f''(x) = 12x^2 - 24x + 12 = 12(x - 1)^2$$

(a) Since $f''(x) = 12(x - 1)^2 \geq 0$, the second derivative is never negative.

For all $x \neq 1$, $f''(x) > 0$, so the function is concave up.

At $x = 1$, the second derivative is zero, but the sign of $f''(x)$ does not change.

Answer

Concave up on

Concave down:

(b) Because the concavity does not change at $x = 1$, there is no inflection point.

Answer

Graph of the function

