

Question 10**Problem:**

Let $f(x) = \frac{x}{x^2+1}$.

- (a) Find the critical points of $f(x)$ on the interval $[-2, 2]$.
- (b) Evaluate the function at the critical points and endpoints.
- (c) Determine the absolute maximum and minimum values of $f(x)$ on the interval.

Question 10 - Solution

We are given:

$$f(x) = \frac{x}{x^2 + 1}$$

(a) Find critical points:

Use quotient rule:

$$f'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

Set $f'(x) = 0$:

$$1 - x^2 = 0 \Rightarrow x = \pm 1$$

(b) Evaluate $f(x)$:

$$f(-2) = \frac{-2}{4+1} = -\frac{2}{5}, \quad f(-1) = \frac{-1}{1+1} = -\frac{1}{2}, \quad f(1) = \frac{1}{1+1} = \frac{1}{2}, \quad f(2) = \frac{2}{4+1} = \frac{2}{5}$$

(c) Absolute extrema:

• Absolute maximum: $f(1) = \frac{1}{2}$

• Absolute minimum: $f(-1) = -\frac{1}{2}$

Graph of $f(x) = \frac{x}{x^2+1}$ on $[-2, 2]$:

