

Question 9**Problem:**

Let $f(x) = x^3 - 3x^2 + 1$.

- (a) Find the critical points of $f(x)$ on the interval $[0, 3]$.
- (b) Evaluate the function at the critical points and endpoints.
- (c) Determine the absolute maximum and minimum values of $f(x)$ on the interval.

Question 9 - Solution

We are given:

$$f(x) = x^3 - 3x^2 + 1$$

(a) Find critical points:

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

Set derivative to zero:

$$3x(x - 2) = 0 \Rightarrow x = 0, x = 2$$

(b) Evaluate $f(x)$ at critical points and endpoints:

$$f(0) = 0 - 0 + 1 = 1$$

$$f(2) = 8 - 12 + 1 = -3$$

$$f(3) = 27 - 27 + 1 = 1$$

(c) Absolute extrema:

- Absolute minimum: $f(2) = -3$
- Absolute maximum: $f(0) = f(3) = 1$

Graph of $f(x) = x^3 - 3x^2 + 1$ on $[0, 3]$:

