

Question 8**Problem:**

Let $f(x) = \frac{x}{x^2+1}$.

- (a) Find all critical points of $f(x)$ on the interval $[-3, 3]$.
- (b) Evaluate $f(x)$ at the critical points and endpoints.
- (c) Determine the absolute maximum and minimum values of $f(x)$ on the given interval.

Question 8 - Solution

We are given:

$$f(x) = \frac{x}{x^2 + 1}$$

(a) Find critical points:

Use the quotient rule:

$$f'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{-x^2 + 1}{(x^2 + 1)^2}$$

Set numerator to 0:

$$-x^2 + 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

(b) Evaluate at endpoints and critical points:

$$f(-3) = \frac{-3}{9 + 1} = -\frac{3}{10}$$

$$f(-1) = \frac{-1}{1 + 1} = -\frac{1}{2}$$

$$f(1) = \frac{1}{1 + 1} = \frac{1}{2}$$

$$f(3) = \frac{3}{9 + 1} = \frac{3}{10}$$

(c) Conclusion:

• Absolute maximum: $f(1) = \frac{1}{2}$

• Absolute minimum: $f(-1) = -\frac{1}{2}$

Graph of $f(x) = \frac{x}{x^2 + 1}$ on $[-3, 3]$:

