

Question 7**Problem:**

A company models its daily profit (in hundreds of dollars) as a function of the number of units produced, using the function:

$$P(x) = -x^3 + 6x^2 + 15$$

where x is the number of units (in hundreds) produced per day, and $x \in [0, 5]$.

- (a) Find all critical points of $P(x)$ on the interval.
- (b) Determine the absolute maximum and minimum profit on the interval $[0, 5]$.

Question 7 - Solution

We are given:

$$P(x) = -x^3 + 6x^2 + 15$$

(a) Find critical points:

Differentiate:

$$P'(x) = -3x^2 + 12x = -3x(x - 4)$$

Set derivative equal to zero:

$$-3x(x - 4) = 0 \Rightarrow x = 0, x = 4$$

These are critical points in the interval $[0, 5]$.

(b) Evaluate endpoints and critical points:

$$P(0) = -0 + 0 + 15 = 15$$

$$P(4) = -64 + 96 + 15 = 47$$

$$P(5) = -125 + 150 + 15 = 40$$

Conclusion:

- Absolute Maximum: $P(4) = 47$
- Absolute Minimum: $P(0) = 15$

Graph of $P(x) = -x^3 + 6x^2 + 15$ on $[0, 5]$:

