

Question 6**Problem:**

Let $f(x) = x^4 - 4x^2$.

- (a) Find the critical points of $f(x)$ in the interval $[-3, 3]$.
- (b) Evaluate the function at the critical points and endpoints.
- (c) Identify the absolute maximum and minimum values of $f(x)$ on the given interval.

Question 6 - Solution

We are given:

$$f(x) = x^4 - 4x^2$$

(a) Find critical points:

$$f'(x) = 4x^3 - 8x = 4x(x^2 - 2)$$

Set derivative to zero:

$$f'(x) = 0 \Rightarrow x = 0, \pm\sqrt{2}$$

(b) Evaluate at critical points and endpoints:

$$f(-3) = (-3)^4 - 4(-3)^2 = 81 - 36 = 45$$

$$f(-\sqrt{2}) = (\sqrt{2})^4 - 4(\sqrt{2})^2 = 4 - 8 = -4$$

$$f(0) = 0^4 - 4(0)^2 = 0$$

$$f(\sqrt{2}) = \text{same as } f(-\sqrt{2}) = -4$$

$$f(3) = 81 - 36 = 45$$

(c) Conclusion:

- Absolute maximum: $f(-3) = f(3) = 45$
- Absolute minimum: $f(\pm\sqrt{2}) = -4$

Graph of $f(x) = x^4 - 4x^2$ on $[-3, 3]$:

